Time-varying overreaction of diagnostic expectations*

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Abstract

Diagnostic expectation is a non-rational expectation framework in which representativeness heuristics distort agents' beliefs and generate overreactions. Most literature assume that severity of representativeness is time-invariant. However, it is not guaranteed. Return on contrarian portfolio, which is profitable if markets overreact, is not always positive. The benefit and unconscious incentives of psychological heuristics would depend on the situations because heuristics are used as mental shortcut to reduce the difficulties with complex decision making. Therefore, we examine the differences of representativeness severity across times. Using TOPIX futures prices as indirect forecast data, we split the samples to five-years subsamples and run SMM to estimate the severity in each subsample. We find that investors significantly overreact to market prices, but the severity is different among subsamples. We find that representativeness is weak when market informativeness is low. We also conduct simulations which allow severity to evolve in like AR(1) process. Our results suggest the positive relationship between persistence/volatility and market informativeness. This paper suggests that the extent of probability distortion caused by representativeness heuristics is also "context dependent."

Keywords: Diagnostic expectations; overreaction; representativeness heuristics

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1 Introduction

It is widely pointed out that many types of economic agents overreact to newly observed information. La Porta (1996) shows that stock market analysts excessively expect firms' future growth. Greenwood and Shleifer (2014) show that investors' expectations tend to be extrapolative. To explain these expectation formation, nonrational expectation models are proposed, such as extrapolative expectations or psychology-based expectations. Bordalo et al. (2018) and Bordalo et al. (2019) propose the diagnostic expectations which is based on the representativeness heuristics. They show that investors distort their beliefs from rational expectation by inflating the probability of representative events or states and diagnostic expectations can explain the credit cycles and overreaction of analysts' forecasts.

Diagnostic expectation shares some features with extrapolative expectation, which agents systematically expect that realized changes keep occurring in the future. In that sense, extrapolative expectation is backward looking whereas diagnostic expectation is forward looking because agents compare the prior probability with posterior probability and overweight probability of representative states of posterior probability. Therefore, the extent of overreaction and overestimated states depend on the prior beliefs and diagnostic expectation is context-dependent framework.

In the diagnostic expectations backed by psychological heuristics, the severity of representativeness heuristics determines the extent of overreaction. Most literature of diagnostic expectations assumes that this severity is time-invariant. It is appropriate to construct theoretical models to demonstrate the impacts of psychological heuristics on agents' expectation formation. After theoretical model is proposed, it is an unexplored filed to examine whether severity is time-invariant.

There are some concerns for time-invariant property. Firstly, contrarian portfolio, which is profitable if stock market overreacts, is not always profitable (Chen and Sauer 1997). If market participants are always influenced by psychological biases at same severity, this portfolio return should have stayed constant. Secondly, psychological heuristic is the mental shortcut for human beings to make decisions quickly to avoid costly complex problems. It is more beneficial to use heuristics when agents face complex problems so that unconscious incentives for heuristic depends on the situations. Therefore, we examine the time variation of psychological influences and investors' overreaction through diagnostic expectations.

We use the TOPIX futures price as market expected value of TOPIX. It is one of the most famous indexed in Japanese market. We split the data to subsamples and run the simulation to estimate the strength of representativeness and macroeconomic structure in each subsample. Our simulation follows the Bordalo et al. (2019) methodology, but target data which simulated time-series data try to fit is replaced to Japanese data. The sample period is from January 1998 to December 2022. Each subsample has five years length so that we have five subsamples and all sample.

Our results show three key findings. First, our Japanese analysis is similar to US analysis of Bordalo et al. (2019). We observe that process of fundamental is persistent and its volatility is higher than transitory shock for all sample. higher volatility of fundamental makes TOPIX prices more informative about fundamental because price changes are more likely to originate from fundamental. Looking at diagnostic parameter, our estimated value is 1.06 and its value in US analysis is 0.9. We confirm that high severity produces significant belief distortions of investors.

Second, our SMM shows the time differences of estimated parameters among subsamples. Subsamples 2008 and 2013 (post (financial) crisis samples) have about 0.64 of persistence parameter and lower volatility of fundamental than transitory shock. The other subsamples have similar values to all sample.

Estimated diagnostic parameters can be divided into two groups. Post financial crisis samples have lower value of severity, which is less than one. Severity of the other subsamples is over 1.3. These results imply that Japanese market overreact to news very strongly when TOPIX prices are informative. In contrast, during 2008 and 2017, TOPIX prices are not as informative as other periods, and Japanese market weakly overreacts to news. Notice that even post crisis samples are less informative, diagnostic Kalman gain is over 0.5, suggesting that investors still react to price movements as if it is informative.

We run Student's t-tests of diagnostic parameter for every subsample pairs. This result confirms that diagnostic parameter in each subsample is statistically significant and reject the null hypothesis that mean value is same.

Third, we run the simulation based on the model which assumes that severity of representaiveness is not time-invariant and follows AR(1) process like fundamental. In former time-invariant analysis, severity parameter is assumed to be fixed in each subsample and we examine whether it is same or different between subsamples. In contrast, this setting allows us to examine the case where diagnostic parameter varies during subsample.

We find that parameters of macroeconomic structures are stable across all time. Volatility of fundamental is always higher than one of transitory shock, making TOPIX prices informative for all subsamples.

We also find that estimated persistence of severity is about 0.4 and less persistent for all sample. In contrast to previous result, post financial crisis samples have close values to all sample. The other subsamples have over 0.67 and more persistent. Estimated volatility of severity process is higher for the other subsamples. These results suggest that investors during post crisis samples are more likely to behave in rational expectation manners. On the contrary, severity of representativeness during the other subsamples gradually changes so that the overreaction lasts and cyclically occurs.

We run subsample pairwise t-test for parameters of severity persistence and volatility. We observe that pairs of (1998, 2003), (1998, 2018) and (2003, 2018) fail to reject the null hypothesis that mean values are same. Pair of post crisis samples is also not statistically significant. These results suggests that there are two regimes of severity process: high persistence and volatility with high informative markets and low persistence and volatility with low informative markets.

We can derive two important implications from our results. Firstly, our results suggest that severity of belief distortions and market informativeness are correlated. Our SMMs suggest that they are positively correlated. It would be conjectured scenario that investors are eager to collect valuable information in uncertain situations and their beliefs are less distorted.

Comparing two SMM results in our analyses, we observe that time-varying model produces lower standard deviations and smaller differences in mean values among subsamples. This suggests that underlying macroeconomic structures are estimated more stable in time-varying model. Stable macroeconomic structure is more likely to generate similar signal surprises in all subsamples. However, we observe that the extent of overreaction is different among subsamples. Given that investor overreaction is derived from belief distortions caused by representativeness and signal surprises, This stableness implies that overreaction is driven more from psychological biases rather than large surprises. It is our second implications.

As additional analyses, we examine the S&P500 futures prices and analysts' forecasts of EPS growth in US firms. In US futures analysis, we find that there are significant overreactions for current news. We observe that fundamental process follows random walk and it is similar among all subsamples. Representativeness severity varies among subsamples; it is around 1.7 for all samples and before 2007, but it drops below 1.0 after 2008. Our result shows that samples after 2008 are characterized with low market informativeness. consistent with main Japanese market analysis.

However, when we run time-varying severity model, different results are obtained. Firstly, samples after 2008 have higher market informativeness than the other samples. Persistence and volatility of severity process is higher during these periods. Market informativeness turns opposite from baseline analysis and the latter relationship is opposite from Japanese analysis.

In case where we use the analysts' forecasts of EPS growth, another different results are observed. This setting is closely related to Bordalo et al. (2019). Firstly, we observe that analysts' forecasts strongly overreact in short term and long term. Secondly, we find that representativeness severity varies among subsamples. Our Student's t-test confirms that samples between 2008 and 2012 have significantly different severity from other periods. During this period, severity is high so that diagnostic distortion is large. We also observe that market informativeness during this period is low. It is opposite relationship from Japanese and US market analyses.

Our time-varying severity model reveals that persistence and volatility of severity are similar among subsamples. It suggests that characteristics of severity do not change substantially among subsamples. The persistence is about 0.5 and the volatility is about 0.1. This process seems to be moderate, not extremely persistent or volatile.

Our main contribution is offering new insights of time-varying characteristics of diagnostic expectations. Our result shows that the severity of representativeness is different across time and varies significantly. It suggests that this varying feature and persistence of severity process might be correlated with market informativeness. Afrouzi et al. (2023) show that agents tend to overreact to further future forecasts of low persistent process. Our implication is close to theirs. In addition, the asymmetry of overreaction impacts between informative period and noisy period can contribute to different behavior of economic agents in boom and burst periods. Maxted (2024) and Krishnamurthy and Li (2020) uses diagnostic expectations and examines the risk tolerance of banks before and after boom phase. Our result enhances their arguments.

Mostly related paper to our analyses is Bianchi et al. (2024). They theoretically develop the smooth diagnostic expectations and show that severity of distortion decreases as the current uncertainty decreases. Our results suggest the relationship between distortion severity and market informativeness. For market analyses using futures prices in Japan and US, our estimated relationship seems to be positive and consistent to the implications of Bianchi et al. (2024).

Notice that diagnostic expectations are also related to selective memory. When decision maker observes new information, states or events whose probability increases the most come to mind first and they are oversampled in mind because of representativeness heuristics. In selective memory literature, people recall the memory from their database and the order of recalling or weights is often decreasing in history. Standard setting assumes that recent events or information is weighted more. Our time-varying property of representativeness strength suggests that the recalling order or weights in mind can vary depending on the context.

Secondly, we add non-US analysis to non-rational expectation literature by using Japanese market data. Most papers in this field conduct empirical analysis with US analyst data. However, psychological heuristics which is a base of diagnostic expectation is not limited to US market conditions, rather it stems from human beings. Applying foreign market analysis enrich the impacts of expectation frameworks based on psychological heuristics.

We use future prices of Japanese stock market index as market participants beliefs. Greenwood and Shleifer (2014) show the survey data of forecasts of many types of US investors. Bordalo et al. (2019) study US analysts' forecasts and explain their systematic forecast errors. Although market price is not direct data of investors' forecasts, it is often argued that market prices reflect aggregated investors' belief in general. Our result suggests that not only professional analysts, but all market participants have overreaction tendency. This is in line with literature which shows that many types of agents hold distorted beliefs (Andre et al. 2022; Bordalo et al. 2020a).

This paper is organized as follows. Section two summarizes the existing literature of diagnostic expectations. Section three describes the framework of diagnostic expectations and section four describes the simulation methodology. Section five shows the result of simulation of time-invariant model and time-varying model. Section six concludes.

2 Prior literature

In the asset pricing field, there are puzzles. The stock returns are more volatile than justified by their dividend flows (Shiller et al. 1981). The required return is higher than expected (Campbell and Shiller 1988). Many researchers propose the variety of required returns; time varying risk preferences (Campbell and Cochrane 1999), and long run risk or disaster risk model (Bansal and Yaron 2004; Barro 2009). Firm characteristics-based factor models are also proposed (Fama and French 2015). These models assume that investors have rational expectations.

In recent years, some studies take another approach: in stead of rational expectations, investors' expectations are shaped by investor sentiment or psychological heuristics. Barberis et al. (1998) propose the investor sentiment driven model and Barberis and Shleifer (2003) consider style investing. Gennaioli and Shleifer (2010) take account of representativeness heuristics, the tendency that people overweight the probability of an event when it is representative of characteristics to its parent population (Kahneman and Tversky 1972).

Based on the psychological background, Bordalo et al. (2018) propose the diagnostic expectations. In this framework, expectations are formed by representativeness heuristics. People overestimate the probability of event which is more likely to occur under the parent population. For example, if investor receive the positive signal, rational investors update their beliefs according to the Bayes' rule, but diagnostic investors, who has diagnostic expectation, overestimate the high productivity state because such state is representative to positive signals.

Therefore, diagnostic investors overreact to positive signals and their expectation becomes more optimistic.

Diagnostic expectations make investor overreact to current news, and their beliefs more volatile than rational expectations. It can explain the survey results of Greenwood and Shleifer (2014) which states that forecasts of many types of agents in US exhibit extrapolative and volatile characteristics.

It is applied to macroeconomic analyses in many papers. Bordalo et al. (2018) show that credit spreads are excessively volatile, overreact to news, and entail predictable reversals. Bordalo et al. (2021b) show that when productivity growth decreases, credit spreads excessively increase. Bordalo et al. (2022) shows that overreaction due to diagnostic expectations generates the predictable boom-bust cycles. Bordalo et al. (2020c) show that in heterogeneous agent model, diagnostic expectations generate individual investors' overreaction, but consensus beliefs exhibit underreaction to news because each investor observes different information. d'Arienzo (2020) applies to affine term-structure model and shows that long-term interest rates are excessively sensitive to news and excessively volatile because it has higher uncertainty than short-term and it makes any signals more informative.

Diagnostic expectations are also used to explain the consumption behavior. L'Huillier et al. (2021) show that consumption also overreacts to supply shocks. Bianchi et al. (2021a) explain the persistent and hum-shaped boom-bust cycle of consumptions. Bianchi et al. (2021b) show that consumption becomes time-inconsistent when reference point is not recent.

Bordalo et al. (2019) apply diagnostic expectations to stock return analysis and show that diagnostic expectations can explain the analysists forecasts' systematic forecast errors and revisions. Bordalo et al. (2020b) show that systematic overreaction generates the price reversals and excess stock market volatility. Bordalo et al. (2021a) show that price overreaction leads to endogenous bubbles and crash.

Krishnamurthy and Li (2020) introduce diagnostic expectations into frictional financial intermediation models and show that banks with diagnostic expectations have higher risk tolerance, decrease the risk premia and increase the credit before crisis. Maxted (2024) shows that disappointment after boom due to excessive optimism of diagnostic expectations make banks tighten their lending, leading to crisis.

Most closely related paper to this analysis is Bianchi et al. (2024). They theoretically develop diagnostic expectation to account for smoothness and show that not only changes in conditional mean, but also in conditional uncertainty affect the belief distortions. They show that the severity of belief distortions decreases as the current uncertainty decreases. If conditional volatility of fundamental decreases but the volatility of transitory shock remains constant, signal-to-noise ratio decreases because signals becomes more noisy. We examine the time-varying property of diagnostic severity and suggest the relationship between severity and market informativeness. Our paper can be seen as a direct empirical analysis of Bianchi et al. (2024).

Most literature applying diagnostic expectations assume that severity of representativeness is time invariant. Several theoretical models are proposed but it is an unexplored to examine whether severity would vary across time. Therefore, we aim to investigate the time variation of psychological influences through diagnostic expectation framework.

We offer new insights to the field of diagnostic expectations and non-rational expectations.

Our results show that influences of representativeness is different across time. Time-varying model in this paper generates more stable estimates of macroeconomic structure, suggesting that time-varying model is more in line with reality. The severity of representativeness and belief distortions seems to be context dependent as well.

We also show that diagnostic severity could be divided to two regimes. One regime is characterized with weak representativeness and low market informativeness. The other is characterized with strong representativeness and high market informativeness. Although our simulations do not reveal any causal relationship, our result suggests that severity of representativeness and market informativeness are correlated. Afrouzi et al. (2023) show the relationship between persistence of underlying process and agents ' overreaction. We provide another finding that investor overreaction is associated with market informativeness and characteristics of underlying process. Given that market informativeness is different between boom and burst phases, our implications enhance the arguments that distorted beliefs leads to excessive optimism in boom period and recession in burst period.

In addition, we add non-US analysis to psychology-based expectation literature by using Japanese market data. Most papers in this field conduct empirical analysis with US data, but psychological heuristics are not exclusive to US markets. Because it stems from human beings, non-US analysis complements this filed in terms of applicability of psychological heuristics.

We use future prices of TOPIX as market participants beliefs. This is motivated by the general idea that market prices reflect the aggregated investors ' beliefs. Several studies point out that forecasts of professional analysts are accompanied by systematical overreaction (La Porta 1996; Bordalo et al. 2019). Our result suggests that not only professional analysts, but all market participants have overreaction tendency. Our paper is in line with literature which shows that beliefs of many types of agents are distorted (Greenwood and Shleifer 2014; Andre et al. 2022; Bordalo et al. 2020a).

This paper is also related to the field of selective memory. In this literature, people have limited capacity for memory so that they recall the memory from their database to form beliefs. Standard setting assumes that recent memory or events are easier to access or weighted more (Bordalo et al. 2020a; Nagel and Xu 2022).

Diagnostic expectations can be interpreted as one form of selective memory; when decision maker observes information, events whose probability increases the most come to mind first and they are oversample in mind. Our result of time-varying severity of representativeness corresponds to time-varying recalling order, weights, or access to memory in selective memory framework. It is suggested that manner of recalling memory is also context dependent.

3 Model: Diagnostic expectation

In this section, we explain diagnostic expectation which is developed by Bordalo et al. (2018) and Bordalo et al. (2019). It is assumed in Bordalo et al. (2019) that the fundamental of economy follows the law of motion

$$f_t = af_{t-1} + \eta_t \tag{1}$$

where $a \in [0,1]$ and $\eta_t \sim N(0, \sigma_\eta^2)$ is an iid normally distributed shock. It is assumed that investors cannot observe f_t directly. Instead, they observe x_t given by

$$x_t = bx_{t-1} + f_t + \epsilon_t \tag{2}$$

where $b \in [0,1]$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ is an iid normally distributed shock. Imposing $b \leq a$ ensures the stationarity.

Rational investors update their beliefs according to the Kalman filter to infer the current fundamental based on the information set after observing signal x_t.

$$E[f_t \mid x_t] = \hat{f}_t = a\hat{f}_{t-1} + K(x_t - bx_{t-1} - a\hat{f}_{t-1}) \tag{3}$$

where $K \equiv (a^2\sigma_f^2 + \sigma_\eta^2)/(a^2\sigma_f^2 + \sigma_\eta^2 + \sigma_\epsilon^2)$ is the signal-to-noise ratio, or called Kalman gain [1].

Bordalo et al. (2018) and Bordalo et al. (2019) propose that investors' beliefs are distorted by the representativeness heuristics. It is argued that agents overestimate the probability of events which is a representative or typical of a parent class (Kahneman and Tversky 1972). Using the measure of the representativeness proposed by Gennaioli and Shleifer (2010), diagnostic expectations are formed by the representativeness-distorted density

$$h^\theta(f_t \mid x_t) = h(f_t|x_t)\left(\frac{h(f_t \mid x_t)}{h(f_t \mid x_{t-1})}\right)^\theta Z \tag{4}$$

where $h(f_t \mid x_t)$ is a rational conditional density, $\theta \geq 0$ is a parameter of representativeness

[1] In steady state, the variance of fundamental is given as the solution to $a^2\sigma_f^4 + \sigma_f^2[\sigma_\eta^2 + (1-a^2)\sigma_\epsilon^2] - \sigma_\eta^2\sigma_\epsilon^2 = 0$.

severity, and Z is a constant ensuring that $h^\theta(f_t \mid x_t)$ integrates to one. With $\theta = 0$, there is no distortions, and it becomes rational density.

In this setup, investors compare the current rational conditional density $h(f_t \mid x_t)$ with past rational conditional density $h(f_t \mid x_{t-1})$ and overestimate (underestimate) the probability if it is more (less) likely to happen under the information set after investors observe the new information. For example, if there is a positive surprise, then the fundamental is more likely to be high, so investors overestimate such states and overreact to positive surprise. Diagnostic expectations are formed by following distorted Kalman filter.

$$E^\theta[f_t \mid x_t] = \hat{f}_t^\theta = a\hat{f}_{t-1} + K(1+\theta)(x_t - bx_{t-1} - a\hat{f}_{t-1}) \tag{5}$$

Bordalo et al. (2019) also show that expected growth of x_t (EG) are characterized by mean reversion as well as fundamental and signal.

$$EG_{t,h} = E^\theta[x_{t+h} - x_t \mid x_t] = -(1-b^h)x_t + a^h \left(\frac{1-(b/a)^h}{1-(b/a)} \right) \hat{f}_t^\theta \tag{6}$$

When there is a positive news, then $\hat{f}_t^\theta$ increases and EG becomes high. Because $\hat{f}_t^\theta \geq \hat{f}_t$ if there is a positive shock, diagnostic investors become more optimistic than rational investors.

4 Simulation

The purpose of this paper is to study the time variation of investor overreaction caused by diagnostic expectations. According to diagnostic expectations, overreaction is driven by the diagnostic parameter (θ) and current signal surprises. We aim to analyze whether the investor overreaction varies and it is caused by the time-varying diagnostic parameter. Overreaction is measured by Coibion and Gorodnichenko (2015) test which is a coefficient of forecast revision to forecast error.

We follow the simulation methodology of Bordalo et al. (2019) to estimate the diagnostic parameter θ and other macroeconomic parameters. They use log EPS as signal of fundamental (x_t) and analyst forecasts of EPS for EG. In contrast, we use the TOPIX market prices as a signal of fundamental and TOPIX futures prices for EG based on the idea that market prices reflect the all market participants' beliefs. TOPIX price data are collected by Nikkei NEEDS Financial QUESTS.

In our analysis, $(a, b, \sigma_\eta, \sigma_\epsilon, \theta)$ are parameters we try to estimate. For every combination of parameters, we simulate a time series of fundamental (f_t) and log TOPIX prices (x_t). Then, we calculate the associated diagnostic expectations about fundamental ($\hat{f}_t^\theta$). Using this expected values, we can compute the the forecast error ($x_{t+h} - x_t - EG_{t,h}$) and forecast revision forecast revision ($EG_{t,h} - EG_{t-k,h}$) for $h = 1, 4$ months. Due to data characteristics, we set the forecast revision lengths to three months($k = 3$). Then, we regress the forecast error on forecast revision to get coefficients $\hat{\gamma}_1$ and $\hat{\gamma}_4$. We also compute the autocorrelation of log TOPIX prices $\hat{\rho}_l = cov(x_t, x_{t-l})/Var(x_t)$ for $l = 1, 2, 3, 4$ months. Four autocorrelation and two regression coefficients are elements of vector of simulated coefficients.

$$v(a, b, \sigma_\eta, \sigma_\epsilon, \theta) = (\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4, \hat{\gamma}_1, \hat{\gamma}_4) \tag{7}$$

After calculating the coefficient vector for every parameter combination, we estimate the best parameters based on the Euclidean distance loss function.

$$l(v) = ||v - \bar{v}|| \tag{8}$$

where $\bar{v}$ is the vector of target coefficients estimated from the original data.

We run this simulation for 30 independent times to get 30 best parameter combinations and analyze the results. We conduct this procedure for all sample period between January 1998 and December 2022. In addition, we also split the sample to 5 years and run the same SMM to obtain the diagnostic parameter for each subsample period. It ensures to verify the difference of investors' overreaction and macroeconomic structure in each subsample.

Bordalo et al. (2019) assume the constant diagnostic parameter, but it is not clear whether it also evolves like fundamental. Chen and Sauer (1997) show that return of contrarian portfolio is not always positive. Contrarian portfolio should be profitable if there is a stock market overreaction. In addition, psychological heuristics are used for mental shortcut to avoid complext problems. The unconscious incentives and benefits may depend on the situation. To consider time-varying case, we assume the time-varying diagnostic parameter and estimate its law of motion.

$$\theta = \max(\theta_{t-1} + \xi_t, 0) \tag{9}$$

where $\xi \sim N(0, \sigma_\xi^2)$ is an iid normally distributed shock.

In this time-varying model, estimated parameters are $(a, b, \sigma_\eta, \sigma_\epsilon, c, \sigma_\xi)$.

5 Results

5.1 Target coefficients

Before estimating the diagnostic parameter, we need a measure of overreaction at each time. Following Bordalo et al. (2019), we run the Coibion and Gorodnichenko (2015) (CG) test.

$$x_{t+h} - x_t - EG_{t,h} = \alpha + \gamma(EG_{t,h} - EG_{t-k,h}) + e_{t+h} \tag{10}$$

CG test shows the extent of investor reaction to observed signals. Positive γ means that investors update their beliefs but it is not enough for realized signal changes, suggesting that investors underreact to observed signals. In contrast, if γ is negative, belief updates are excessive compared to realized changes. Since diagnostic investors systematically overreact to current surprises, the expected sign of γ is negative for $\theta > 0$ and zero for $\theta = 0$ (corresponding to rational investors).

We use the price of futures whose contract month is one month ahead as the expected value of TOPIX price one month ahead. Because contract month of TOPIX futures is every three months, we fix the revision interval (k) to three months.

Table 1 shows the result of CC regression for all samples. We find that there is a negative coefficient of forecast revision over past three months to forecast error over one month. It is statistically significant. This is also found in relation of forecast error over four months. Except for two and three months forecast error, positive surprises drive higher growth expectations, but it is significantly higher than realized growth on average.

5.2 Fixed diagnostic parameter

We run CG regression and calculate the autocorrelation of log TOPIX prices for each subsamples. Target vectors in our SMM($\bar{v}$) is summarized in table 2. This table shows that autocorrelation of one month (ρ_1) is about 0.83-0.97. Autocorrelation of log TOPIX decreases after the global financial crisis. Notice that the length of subsample is 5 years. Subsample of 2008 and 2018 contains the global financial crisis and COVID outbreak. The coefficient of forecast revision to forecast error in one month (γ_1) is positive for these subsamples, implying that initial reaction during these uncertain situations was not enough so that there were subsequent price adjustments.

Parameters in our SMM are defined by $a, b \in [0,1]$, $\sigma_\eta, \sigma_\xi \in [0, 0.5]$, and $\theta \in [0,2]$. a,b, and θ are defined in step of 0.1 and σ_η and σ_ϵ are defined in step of 0.05.

Table 3 shows the mean and standard deviation of estimates of model parameters for all sample and subsamples. Estimated persistence parameter of fundamental (a) is 0.87 for all sample, but it drops to 0.65 for 2008 and 2013 subsamples. We call these subsamples post crisis samples because they are after global financial crisis. These subsamples have higher standard deviations for a than the other subsamples, implying that there would be difficulties for simulation during these periods. The volatility of shock on fundamental (σ_η) is higher than the volatility of transitory shock (σ_ϵ) for all sample, but it is less for post crisis samples. Lower volatility of transitory shock implies that the change of signals is more likely to originate from the unobserved fundamental. Therefore, we observe that Kalman gain (K) is above 0.5 for all sample, but not for post crisis samples.

Most interested parameter in this analysis is the diagnostic parameter, θ. There are two key findings. First, our estimates show the strong diagnostic effects. θ is about or over one for most of the samples except 2013 subsample which is the lowest value of 0.58. Since diagnostic investors react to surprises by $(1+\theta)K$, if θ is one, then they react to news as if signal-to-noise ratio is doubled. For example, Kalman gain is 0.7 for all sample, but diagnostic Kalman gain is 1.45, suggesting that they excessively overreact to new surprises.

When Kalman gain and diagnostic parameter are small, impact of diagnostic expectation is not as huge as to cause the excessive overreaction because small Kalman gain implies the uninformative signal and small θ induces relatively small overreaction. However, our diagnostic Kalman gain for post crisis samples, which have small Kalman gain and θ, are above 0.5. It suggests that even observed signal has more noise component than fundamental, diagnostic investors systematically think it is informative and try to extract the information about fundamental.

Second, diagnostic parameters vary significantly across time. The lowest value is 0.58 and the highest value is 1.83, more than 3 times of lowest value. Diagnostic parameter decreases in post crisis samples, but it recovers to 1.34 in 2018 subsample. It does not show the monotonic trend. Even though the standard deviation of θ is high, this variation suggests that the severity of representativeness which is the source of diagnostic expectations would not stay constant and vary in time.

To examine the differences of diagnostic severity, we conduct a Student's t-test for every pair of subsamples. Table 4 shows the difference of mean values in subsamples and p values. All pairs of subsamples except pair of (all, 2018) are statistically significant, rejecting the null hypothesis that mean value of θ is same. This result confirms the differences of diagnostic severity to cause the overreaction in diagnostic expectations framework. Post crisis samples are not only different from other samples, but also different each other.

Interesting findings in table 3 is the relationship between signal informativeness and the representativeness severity. We observe that during post crisis samples, Kalman gain is low and the diagnostic severity is also low. The other subsamples have higher informativeness and severity. Our result suggests that signal informativeness and diagnostic severity are correlated or there are hidden variables which influence the them.

We also observe that macroeconomic parameters such as a, b, σ_η and σ_ϵ have relatively small standard deviation in subsamples except post crisis samples. However, CG regression coefficients and estimated diagnostic parameters have significant differences. It suggests that different extent of overreaction is caused by the different severity of diagnostic expectations, not originated from the signal surprises.

5.3 Time-varying diagnostic parameter

In previous analysis, we observe that θ is different in each subsample. Its variation seems not to be monotonical change. Diagnostic severity in each subsample is statistically different from one in other subsamples. In this section, we study the case where θ is also formed by AR(1).

In this setup, we assume that θ follows the equation (9) and estimated parameters are $(a, b, c, \sigma_\eta, \sigma_\epsilon, \sigma_\xi)$.

As in the previous analysis, we simulate a time series of fundamental (f_t), log TOPIX prices (x_t), and the diagnostic parameter (θ_t). Then we compute the autocorrelation of

TOPIX prices and coefficients of forecast revision to forecast errors. Parameters of θ_t are defined by $c \in [0,1]$ and $\sigma_\xi \in [0,0.5]$ with step of 0.1 and 0.025. We pick the parameter combination which minimizes the Euclidean loss function. Notice that the target vector is same to previous one in table 2. We set the initial value of c is 0, corresponding to the rational expectation case.

Table 5 shows the mean and standard deviation of estimated parameters across 30 independent simulations. In this analysis, fundamental is estimated more persistent than previous one. Estimates of a are around 0.9 and do not vary in subsamples. They have significantly smaller standard deviation than previous results. Mean estimates of b vary in subsamples, but they also have lower standard deviation. We also observe that volatility terms have stable estimates than time-invariant model. It suggests that macroeconomic structure does not vary significantly during our sample periods.

Looking at volatility of shock on fundamental (σ_η), it always has higher than the volatility of transitory shock (σ_ϵ). Therefore, Kalman gain is above 0.5 for all time and the observed signal is always informative.

The most interested parameters in this analysis are the persistence (c) and the volatility (σ_ξ) of θ process. we find the mixed result in terms of persistence. Estimated parameter of persistence (c) for all sample is 0.39, which is less persistent. However, it is higher in all subsamples than all sample. Post crisis samples have around 0.47, and the other subsamples have over 0.67. Notice that all subsamples have similar standard deviations, but the level of standard deviation is higher than other parameters like a or b. Higher persistence implies that the severity of representativeness gradually changes so that the overreaction lasts and cyclically occurs. If persistence of θ is low, people suddenly overreact to news or turn back to rational expectations.

Similar to previous section, we observe that post crisis samples are characterized with low persistence of θ and relatively low Kalman gain, low informativeness of signals. Given that θ would easily go to 0 in low persistent process, it is suggested that investors are more likely to behave in rational expectation manners in the uncertain situation.

Our estimated volatility of severity process (σ_ξ) is 0.21 for all sample, varying between 0.2 and 0.38 in subsamples. Volatility of θ is around 0.22 in post crisis samples whereas it is around 0.37 in the other subsamples. Given that diagnostic Kalman gain is calculated by $(1+\theta)K$, average change of overreaction is about 20%.

We also observe that estimated σ_ξ is high when Kalman gain is high. If σ_ξ correlates to Kalman gain positively, overreaction due to the diagnostic expectation are more likely to occur when the signal is more informative and the extent of overreaction changes more largely.

Table 6 shows the result of Student's t-test of estimated c and table 7 shows the result of σ_ξ. We observe that pairs of (1998,2003), (1998,2018) and (2010,2018) fail to reject the null hypothesis that mean values are same. Pair of post crisis samples is also not statistically significant. These results suggests that there are two regimes in severity process: high persistence state with high signal informativeness and low persistence state with low signal informativeness. Looking at all sample, they are statistically insignificant to post crisis samples, suggesting that all sample characteristics are closer to post crisis samples than the other subsamples.

In summary, time-varying severity model shows the standard deviations of a, b, σ_η and

σ_ϵ are lower than time-invariant model, so that time-varying model is characterized with the stability of macroeconomic structure. Stable macroeconomic model suggests that signal surprises in each subsample are not significantly different. It supports that the severity of representativeness is the source of investors' overreaction. Subsample pairwise t-test suggests the two regimes in severity process and highlights the time-invariant property of diagnostic severity.

6 Additional analyses

6.1 S&P 500 analysis

In this section, we use the S&P 500 index instead of TOPIX. This analysis contributes to main analysis in terms of robustness. We collect the S&P 500 futures data from Refinitiv Eikon. Same SMM methodology applies to this analysis. We use log S&P 500 index as a signal of fundamental (x_t) and futures prices for expected values. Sample starts 1998 to 2022, same to Japanese market analysis. Each subsample has five years length.

Table 8 shows the CG regression coefficients of price differences between realized returns and expected returns on expected returns changes for all samples. Notice that price data are monthly but returns are annualized. We observe that if period of forecast revision (k) is three, all coefficients are negative and statistically significant. Coefficient values of k=3 have highest magnitude for every forecast error length (h). Possible explanation is that contract month of S&P 500 futures is three month.

As target vector for SMM in US futures analysis, we fix k=3 and calculate the CG coefficients for $h = 1, 3$ months. Autocorrelation is calculated based on $l = 1, 2, 3, 4$ months. Table 9 summarizes the regression coefficients and autocorrelation of target vectors. Autocorrelations exhibit similar trends except subsample 2013. This subsample has positive autocorrelations. CG regression coefficients are negative except for subsample 2008. Notice that large negative coefficients implies that markets strongly overreact to current surprises. Therefore our CG test suggests that market overreact except subsample 2008, which is after global financial crisis.

Table 10 shows the SMM result of time-invariant model for all samples and every subsamples. We observe that estimated persistence of fundamental process is one, meaning that fundamental follows random walk. Volatility of fundamental is ranged between 0.21 and 0.28. persistence of signal is about 0.65 for all samples, subsample 1998 and 2003 whereas other subsamples have weaker persistence, especially subsample 2008. Volatility of transitory shock is also higher after 2008. Subsample 2008 and 2013 have Kalman gain less than 0.5, suggesting that signals are noisy.

US futures analysis exhibits the time-varying property of diagnostic parameter. It is about 1.7 for all samples and subsample 1998 and 2003. After 2008, it drops less than one. However, it does not show clear relationship between severity of representativeness and market informativeness which is observed in Japanese market analysis. Post crisis samples (subsample 2008 and 2013) are characterized with low market informativeness, but two smallest severities are subsample 2013 and 2018.

Table 11 shows the Student's t-test of θ for every subsample pair. There are five subsample pairs which are statistically insignificant and fail to reject null hypothesis that mean values are same. These results imply that there are two groups: one includes all samples, subsample 1998 and 2003, the other consists of subsamples after 2008. Former group has relatively higher market informativeness and representativeness severity. During these periods, diagnostic Kalman gain is over 1.5, suggesting that investors strongly overreact to newly arriving news.

Table 12 shows the SMM result of time-varying model. Similar to time-invariant model, fundamental seems to follow random walk. For signal process, persistence is stable around 0.6 except subsample 2008 which has 0.32. Surprisingly, market informativeness represented by Kalman gain has opposite result. Former time-invariant model shows that subsamples after 2008 have lower informativeness than other periods. These periods have higher persistence of representativeness severity (c) and lower volatility (σ_ξ) than all samples and subsample 1998 and 2003. Table 13 shows the Student's t-test for c and σ_ξ. T-test of c suggests that there are two regimes: one consists of all samples, subsample 1998, and 2003, and the other is after 2008. The relationship between market informativeness and severity in Japanese analysis and US analysis is summarized in table 14.

6.2 Analysts' forecasts of US firms' EPS growth

In this section, we focus on the analysts' forecasts of EPS growth in US firms. We collect US data from Refinitiv Eikon. SMM methodology follows the previous section. We use log value of EPS as signals(x_t) and mean value of analysts' forecasts of EPS as EG. Parameter definition is same to previous one. Sample starts 1988 to 2022. We split subsamples which has five years length so that we have one full sample and six subsamples. Notice that analysts' forecasts and firm EPS are announced in every quarter.

Table 15 shows the CG regression of analysts forecasts and realized EPS growth for all samples. EPS growth is calculated by $(EPS_{t+h}/EPS_t)^{1/(h/4)}$, which is annualized. We observe that very short-term CG regression coefficients, for example (k=1, h=1), have significantly negative values. It suggests that analysts strongly overreact to newly observed signals in short term. When forecast length is less than one year and revision period is two or three quarters, this regression coefficients turn to positive. This trend is also observed in previous analysis which uses the Japanese market data.

We also observe that coefficients are negative and statistically significant in yearly base such as k=4,8 and h=4,8 quarters. CG regression with large k and small h are mostly insignificant. This might be caused by database restrictions because long-term analysts' forecasts are less stored than short-term in our database.

As target vector for SMM in US EPS analysis, we adopt yearly base CG regression. We use one and two years for CG regression. One coefficient is obtained from one year forecast errors and forecast revision; the other is obtained from two years set. As autocorrelation, we calculate $l = 1, 2, 3, 4$ quarters. Table 16 summarizes the regression coefficients and autocorrelation as target vectors in SMM. We observe that subsample 1988 and 1993 have high autocorrelation for one quarter. This might be one reason why CG coefficients for one year in these samples have positive values. Looking at one year coefficients (γ_1), we find that subsample 2003 and 2008 also have positive values. It suggests that analysts' reaction during these periods is not enough and they underreact to signals. Except subsample 2003, two year coefficients (γ_2) are negative, suggesting that they overreact in long-term forecasts.

Table 17 shows the SMM result of time-invariant model for all samples and every subsamples. Our result exhibits very persistent fundamental because mean value of estimated a is almost one and standard deviations are very small. However, signals are not persistent. It may make volatility of fundamental process very small and volatility of transitory shock larger than fundamental one. When transitory shock volatility is larger, signal contains more noise and becomes less informative. Therefore, our estimated Kalman gain for all sample is very small.

Most interested parameter of θ in this analysis is large value for all samples and varies among subsamples. All sample has 1.15 of θ, smallest value is 0.72 in subsample 2013 and highest value is 1.77 in subsample 2008. In this analysis, subsample 1993 has second smallest value of θ, but subsample 1993 and 2013 exhibits two largest Kalman gain. Subsample 2008 has largest θ and third smallest Kalman gain. subsample 2018 has second largest θ and second smallest Kalman gain. Our results suggest opposite relationship between diagnostic severity and signal informativeness from market analysis in Japan and US. When signal is (un)informative, analysts' forecasts are less(more) distorted from rational expectations.

Table 18 shows the Student's t-test of θ for every pair of subsamples. We observe that only subsample 2008, which is directly after the global financial crisis, are statistically different from all other subsamples. This subsample has significantly higher value of θ, suggesting that market condition would influence the severity of representativeness.

Table 19 shows the SMM result of time-varying model. Similar to main analysis, we find that time-varying model generate stable parameters of macroeconomic structure. We observe that persistence of fundamental (a) is almost 1 in all subsamples and further smaller values of standard deviation than time-invariant model. Similar to previous analysis, estimated Kalman gain in all sample is small because volatility of fundamental is smaller than one of transitory shock.

Focusing on the parameters of severity process, we observe that persistence and volatility parameters do not exhibit significant changes among subsamples. Persistence (c) is about 0.5 and volatility (σ_ξ) is about 0.3. Not like Japanese cases where severity process seems to have two regimes depending on market informativeness, estimated parameters in subsamples are close each other. Relationship with market informativeness is summarized in table 14. Table 20 and table 21 shows the t-test for c and σ_ξ. We find that subsample pairs of (all,1993) and (1988,1993) for σ_ξ are only statistically significant, rejecting the null hypothesis that mean values are same. This result suggests that there would be one severity process which is consistent to all subsamples.

7 Conclusion

Under diagnostic expectations, investors overestimate the representative states and overreact to observed signals. Their beliefs are distorted because of representativeness heuristics. Most literature assumes that the severity of representativeness is time invariant. However, unconscious incentives for psychological heuristics, which is the mental shortcut, depends on the situation. In addition, profits of contrarian portfolio are not always positive, suggesting that stock market overreaction is not time invariant.

We run SMM to investigate the time differences of representativeness severity across subsamples. We find that representativeness is weak between 2008 and 2017. Subsamples pairwise t-tests confirm that diagnostic severity is significantly different each other. We also find that market informativeness during these periods is lower than the other periods. Even Kalman gain in these periods is low, diagnostic expectations inflate the informativeness, implying that investors react to information as if it is informative and overreaction occurs.

We also conduct time-varying severity analysis. We observe that this model produces more stable parameters of macroeconomic structures. Our results suggests that there are two regimes in severity process. The estimated persistence and volatility of severity process is low between 2008 and 2017. The other periods have higher persistence and volatility. Subsample pairwise t-tests confirms the differences of two regimes.

Our analysis suggests that the severity of representativeness heuristics is correlated to market informativeness. Especially, when market situation is uncertain, market informativeness is low and strength of representativeness is weak, or severity process is less persistent. It suggests that investors are more likely to behave in line with rational expectations when they seek precise information. We offer new insights for investor sentiment analysis and future research should consider this relationship.

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Table 1. Result of Coibion and Gorodnichenko (2015) test.

	coeffs	stand. err	p value
h=1	-2.127	0.614	0.001
h=2	1.597	1.733	0.360
h=3	0.185	1.608	0.909
h=4	-0.328	0.103	0.002
h=5	-0.191	0.104	0.071
h=6	-0.274	0.107	0.013

This table shows the regression coefficient of forecast errors on forecast revision. We fix the forecast revision intercal to 3 month.

Table 2. Target coefficient values.

	all sample	1998	2003	2008	2013	2018
ρ_1	0.97	0.928	0.948	0.864	0.832	0.859
ρ_2	0.944	0.856	0.897	0.732	0.718	0.786
ρ_3	0.916	0.771	0.837	0.63	0.615	0.685
ρ_4	0.887	0.69	0.769	0.517	0.553	0.631
γ_1	-2.177	-4.839	-6.668	2.669	-4.346	2.95
γ_4	-0.532	-0.301	0.486	-11.537	-3.883	-1.403

ρ_l is a autocorrelation of TOPIX for $l = 1, 2, 3, 4$ months. γ_h is a regression coefficient of CG test for $h = 1, 4$ months. These are target vectors for simulations.

Table 3. Estimated parameters of time-invariant severity model.

	all sample	1998	2003	2008	2013	2018
θ	1.063	1.833	1.633	0.983	0.583	1.34
	(0.798)	(0.188)	(0.47)	(0.597)	(0.484)	(0.521)
a	0.873	0.86	0.857	0.653	0.637	0.893
	(0.052)	(0.05)	(0.05)	(0.296)	(0.318)	(0.025)
b	0.47	0.68	0.687	0.363	0.123	0.537
	(0.274)	(0.061)	(0.09)	(0.243)	(0.05)	(0.138)
σ_η	0.344	0.42	0.387	0.148	0.213	0.408
	(0.11)	(0.08)	(0.096)	(0.12)	(0.16)	(0.098)
σ_ϵ	0.255	0.158	0.158	0.321	0.352	0.217
	(0.183)	(0.135)	(0.125)	(0.135)	(0.112)	(0.137)
Kalman gain(K)	0.704	0.885	0.869	0.256	0.347	0.807
diagnostic K	1.452	2.508	2.289	0.507	0.55	1.889

This table shows the mean value and standard deviation of estimated coefficients. Standard deviation is inside the brackets. Kalman gain is calculated by $K \equiv (a^2\sigma_f^2 + \sigma_\eta^2)/(a^2\sigma_f^2 + \sigma_\eta^2 + \sigma_\epsilon^2)$ and diagnostic Kalman gain (diagnostic K) is calculated by $(1+\theta)K$. If Kalman gain or diagnostic Kalman gain is over one, investors overreact observed signals.

Table 4. Pairwise t-test of estimated θ.

	all sample	1998	2003	2008	2013	2018
all	0.0					
	(1.0)					
1998	0.77	0.0				
	(0.0)	(1.0)				
2003	0.57	-0.2	0.0			
	(0.0013)	(0.0346)	(1.0)			
2008	-0.08	-0.85	-0.65	0.0		
	(0.6617)	(0.0)	(0.0)	(1.0)		
2013	-0.48	-1.25	-1.05	-0.4	0.0	
	(0.0066)	(0.0)	(0.0)	(0.0061)	(1.0)	
2018	0.277	-0.493	-0.293	0.357	0.757	0.0
	(0.1171)	(0.0)	(0.0257)	(0.0167)	(0.0)	(1.0)

This table shows the difference of mean value of θ in each subsample and p value of t-tests in parentheses. Null hypothesis is that both subsamples have same mean value.

Table 5. Estimated parameters of time-varying severity model.

	all sample	1998	2003	2008	2013	2018
c	0.393	0.74	0.767	0.463	0.483	0.673
	(0.2)	(0.222)	(0.225)	(0.267)	(0.248)	(0.223)
σ_ξ	0.212	0.383	0.368	0.237	0.205	0.375
	(0.127)	(0.137)	(0.123)	(0.148)	(0.119)	(0.106)
a	0.84	0.897	0.9	0.9	0.9	0.877
	(0.056)	(0.018)	(0.0)	(0.0)	(0.0)	(0.043)
b	0.207	0.67	0.693	0.41	0.163	0.48
	(0.146)	(0.121)	(0.105)	(0.099)	(0.085)	(0.089)
σ_η	0.448	0.393	0.4	0.373	0.413	0.48
	(0.061)	(0.121)	(0.086)	(0.094)	(0.082)	(0.034)
σ_ϵ	0.45	0.25	0.212	0.237	0.297	0.172
	(0.059)	(0.15)	(0.139)	(0.127)	(0.106)	(0.132)
Kalman gain(K)	0.54	0.808	0.813	0.678	0.657	0.717

This table shows the mean value and standard deviation of estimated coefficients obtained by SMM with time-varying θ model. Standard deviation is inside the brackets.

Table 6. Pairwise t-test of estimated c.

	all sample	1998	2003	2008	2013	2018
all	0.0					
	(1.0)					
1998	0.347	0.0				
	(0.0)	(1.0)				
2003	0.374	0.027	0.0			
	(0.0)	(0.6458)	(1.0)			
2008	0.07	-0.277	-0.304	0.0		
	(0.2552)	(0.0001)	(0.0)	(1.0)		
2013	0.09	-0.257	-0.284	0.02	0.0	
	(0.127)	(0.0001)	(0.0)	(0.7648)	(1.0)	
2018	0.28	-0.067	-0.094	0.21	0.19	0.0
	(0.0)	(0.2506)	(0.1117)	(0.0016)	(0.0028)	(1.0)

This table shows the difference of mean value of c in each subsample and p value of t-tests in parentheses. Null hypothesis is that both subsamples have same mean value.

Table 7. Pairwise t-test of estimated σ_ξ.

	all sample	1998	2003	2008	2013	2018
all	0.0					
	(1.0)					
1998	0.171	0.0				
	(0.0)	(1.0)				
2003	0.156	-0.015	0.0			
	(0.0)	(0.6573)	(1.0)			
2008	0.025	-0.146	-0.131	0.0		
	(0.4854)	(0.0002)	(0.0004)	(1.0)		
2013	-0.007	-0.178	-0.163	-0.032	0.0	
	(0.8347)	(0.0)	(0.0)	(0.365)	(1.0)	
2018	0.163	-0.008	0.007	0.138	0.17	0.0
	(0.0)	(0.7937)	(0.823)	(0.0001)	(0.0)	(1.0)

This table shows the difference of mean value of σ_ξ in each subsample and p value of t-tests in parentheses. Null hypothesis is that both subsamples have same mean value.

Table 8. Result of CG test for S&P500 futures.

	h=1	h=2	h=3	h=4
k=1	-1.427	-1.032	-0.687	-0.143
	(0.035)	(0.263)	(0.0)	(0.328)
k=2	-2.115	-1.426	-0.551	-0.223
	(0.002)	(0.124)	(0.0)	(0.038)
k=3	-4.002	-3.155	-0.486	-0.32
	(0.001)	(0.026)	(0.0)	(0.001)

This table shows the regression coefficient of forecast errors on forecast revision. We fix the forecast revision intercal to 3 month.

Table 9. Target coefficient values for US index analysis.

	all sample	1998	2003	2008	2013	2018
ρ_1	0.99	0.949	0.976	0.937	1.003	0.971
ρ_2	0.979	0.899	0.92	0.862	1.008	0.947
ρ_3	0.969	0.858	0.875	0.793	1.014	0.922
ρ_4	0.958	0.811	0.865	0.719	1.022	0.902
γ_1	-4.002	-3.947	-7.478	2.466	-11.454	-2.997
γ_3	-0.486	-0.601	-0.141	-4.977	-13.844	-10.021

ρ_l is a autocorrelation of S&P500 index for $l = 1, 2, 3, 4$ months. γ_h is a regression coefficient of CG test for $h = 1, 3$ months and $k = 3$ months. These are target vectors for simulations.

Table 10. Estimated parameters of time-invariant severity model of US index analysis.

	all sample	1998	2003	2008	2013	2018
θ	1.752	1.745	1.732	0.965	0.718	0.785
	(0.418)	(0.44)	(0.471)	(0.611)	(0.587)	(0.589)
a	1.0	1.0	1.0	1.0	1.0	1.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
b	0.655	0.672	0.64	0.312	0.442	0.432
	(0.206)	(0.189)	(0.21)	(0.174)	(0.242)	(0.193)
σ_η	0.212	0.208	0.278	0.242	0.264	0.281
	(0.093)	(0.1)	(0.085)	(0.094)	(0.113)	(0.107)
σ_ϵ	0.181	0.186	0.211	0.274	0.291	0.274
	(0.11)	(0.116)	(0.107)	(0.096)	(0.104)	(0.12)
Kalman gain (K)	0.582	0.557	0.636	0.447	0.459	0.52
diagnostic K	1.601	1.529	1.738	0.879	0.789	0.929

This table shows the mean value and standard deviation of estimated coefficients. Standard deviation is inside the brackets. Kalman gain is calculated by $K \equiv (a^2\sigma_f^2 + \sigma_\eta^2)/(a^2\sigma_f^2 + \sigma_\eta^2 + \sigma_\epsilon^2)$ and diagnostic Kalman gain (diagnostic K) is calculated by $(1+\theta)K$. If Kalman gain or diagnostic Kalman gain is over one, investors overreact observed signals.

Table 11. Pairwise t-test of estimated θ for US index analysis.

	all sample	1998	2003	2008	2013	2018
all	0.0					
	(1.0)					
1998	-0.01	0.0				
	(0.938)	(1.0)				
2003	-0.02	-0.01	0.0			
	(0.841)	(0.903)	(1.0)			
2008	-0.79	-0.78	-0.77	0.0		
	(0.0)	(0.0)	(0.0)	(1.0)		
2013	-1.03	-1.03	-1.01	-0.25	0.0	
	(0.0)	(0.0)	(0.0)	(0.069)	(1.0)	
2018	-0.97	-0.96	-0.95	-0.18	0.07	0.0
	(0.0)	(0.0)	(0.0)	(0.184)	(0.609)	(1.0)

This table shows the difference of mean value of θ in each subsample and p value of t-tests in parentheses. Null hypothesis is that both subsamples have same mean value.

Table 12. Estimated parameters of time-varying severity model of US index analysis.

	all sample	1998	2003	2008	2013	2018
c	0.833	0.813	0.877	0.587	0.46	0.467
	(0.244)	(0.27)	(0.234)	(0.281)	(0.262)	(0.286)
σ_ξ	0.352	0.318	0.338	0.258	0.23	0.283
	(0.138)	(0.139)	(0.119)	(0.162)	(0.137)	(0.157)
a	1.0	1.0	1.0	1.0	1.0	1.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
b	0.643	0.633	0.673	0.317	0.6	0.56
	(0.218)	(0.214)	(0.205)	(0.184)	(0.178)	(0.218)
σ_η	0.197	0.205	0.248	0.287	0.308	0.285
	(0.107)	(0.109)	(0.1)	(0.086)	(0.091)	(0.104)
σ_ϵ	0.208	0.215	0.207	0.247	0.262	0.235
	(0.12)	(0.124)	(0.102)	(0.12)	(0.096)	(0.123)
Kalman gain(K)	0.476	0.481	0.594	0.579	0.587	0.599

This table shows the mean value and standard deviation of estimated coefficients obtained by SMM with time-varying θ model. Standard deviation is inside the brackets.

Table 13. Pairwise t-test of estimated c and σ_ξ for US index analysis

(a) Persitence parameter c

	all sample	1998	2003	2008	2013	2018
all	0.0					
	(1.0)					
1998	-0.02	0.0				
	(0.765)	(1.0)				
2003	0.04	0.06	0.0			
	(0.486)	(0.336)	(1.0)			
2008	-0.25	-0.23	-0.29	0.0		
	(0.001)	(0.002)	(0.0)	(1.0)		
2013	-0.37	-0.35	-0.42	-0.13	0.0	
	(0.0)	(0.0)	(0.0)	(0.076)	(1.0)	
2018	-0.37	-0.35	-0.41	-0.12	0.01	0.0
	(0.0)	(0.0)	(0.0)	(0.107)	(0.925)	(1.0)

(b) Volatility parameter σ_ξ

	all sample	1998	2003	2008	2013	2018
all	0.0					
	(1.0)					
1998	-0.03	0.0				
	(0.356)	(1.0)				
2003	-0.01	0.02	0.0			
	(0.69)	(0.553)	(1.0)			
2008	-0.09	-0.06	-0.08	0.0		
	(0.02)	(0.13)	(0.034)	(1.0)		
2013	-0.12	-0.09	-0.11	-0.03	0.0	
	(0.001)	(0.016)	(0.002)	(0.468)	(1.0)	
2018	-0.07	-0.03	-0.05	0.03	0.05	0.0
	(0.078)	(0.364)	(0.132)	(0.546)	(0.166)	(1.0)

This table shows the difference of mean value of c and σ_ξ in each subsample and p value of t-tests in parentheses. Null hypothesis is that both subsamples have same mean value.

Table 14. Relationship with market informativeness and severity parameters.

	JPN market	US market	US EPS
θ	positive	positive	negative
c	positive	negative	-
σ_ξ	positive	negative	-

This table shows the relationship between market informativeness and parameters of representativeness severity.

Table 15. Result of CG test for analysts' forecast of US firms' EPS.

	h=1	h=2	h=3	h=4	h=5	h=8	h=12	h=16	h=20
k=1	-17.542	0.071	0.014	-0.224	-0.097	-0.153	-0.036	-0.346	-0.303
	(0.0)	(0.658)	(0.115)	(0.0)	(0.0)	(0.0)	(0.309)	(0.0)	(0.026)
k=2	-4.69	0.1	-0.039	-0.216	-0.15	-0.153	-0.043	-0.145	0.145
	(0.0)	(0.498)	(0.0)	(0.0)	(0.0)	(0.0)	(0.258)	(0.0)	(0.18)
k=3	0.0	0.0	-0.093	-0.054	-0.222	-0.158	-0.145	-0.083	-0.047
	(0.995)	(0.998)	(0.0)	(0.0)	(0.0)	(0.0)	(0.001)	(0.013)	(0.746)
k=4	0.0	0.0	-0.227	-0.017	-0.293	-0.241	-0.21	-0.102	0.261
	(0.997)	(0.998)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.003)	(0.215)
k=8	0.0	0.0	-0.194	-0.005	-0.205	-0.176	-0.153	-0.068	0.497
	(1.0)	(0.998)	(0.0)	(0.004)	(0.0)	(0.0)	(0.0)	(0.161)	(0.052)
k=12	0.0	0.0	-0.022	0.0	-0.151	-0.055	-0.056	-0.506	-0.235
	(1.0)	(1.0)	(0.013)	(0.99)	(0.0)	(0.0)	(0.002)	(0.0)	(0.373)
k=16	0.0	0.0	-0.017	0.0	-0.095	-0.045	-0.076	-0.459	-0.453
	(1.0)	(1.0)	(0.045)	(0.767)	(0.0)	(0.0)	(0.0)	(0.0)	(0.001)
k=20	0.0	0.0	0.01	0.0	-0.118	-0.039	-0.082	-0.337	-0.332
	(1.0)	(0.999)	(0.281)	(0.622)	(0.0)	(0.0)	(0.0)	(0.0)	(0.03)

This table shows the regression coefficient of forecast errors on forecast revision. We fix the forecast revision intercal to 3 month.

Table 16. Target coefficient values for US index analysis.

	all sample	1988	1993	1998	2003	2008	2013	2018
ρ_1	0.858	1.025	1.002	0.764	0.951	0.835	0.879	0.783
ρ_2	0.819	0.894	1.052	0.727	0.913	0.778	0.829	0.736
ρ_3	0.79	0.934	0.987	0.746	0.854	0.743	0.801	0.728
ρ_4	0.778	0.99	1.016	0.73	0.869	0.744	0.788	0.714
γ_1	-0.017	0.2	0.288	-0.103	0.036	0.233	-0.088	-0.02
γ_2	-0.176	-0.12	-1.486	-0.025	0.002	-0.164	-0.06	-0.305

ρ_l is a autocorrelation of EPS growth for $l = 1, 2, 3, 4$ quarters. γ_i is a regression coefficient of CG test for $h = k = i$ years. These are target vectors for simulations.

Table 17. Estimated parameters of time-invariant severity model of US EPS analysis.

	all sample	1988	1993	1998	2003	2008	2013	2018
θ	1.152	1.365	0.902	0.992	0.972	1.77	0.718	1.492
	(0.618)	(0.644)	(0.685)	(0.581)	(0.787)	(0.672)	(0.497)	(0.702)
a	1.0	1.0	1.0	0.945	0.93	0.995	0.9	1.0
	(0.0)	(0.0)	(0.0)	(0.05)	(0.046)	(0.022)	(0.0)	(0.0)
b	0.225	0.325	0.575	0.138	0.628	0.228	0.378	0.235
	(0.101)	(0.172)	(0.212)	(0.07)	(0.23)	(0.16)	(0.246)	(0.123)
σ_η	0.052	0.071	0.321	0.106	0.138	0.094	0.098	0.072
	(0.011)	(0.054)	(0.082)	(0.062)	(0.11)	(0.043)	(0.041)	(0.028)
σ_ϵ	0.296	0.216	0.238	0.225	0.186	0.341	0.098	0.375
	(0.08)	(0.098)	(0.118)	(0.08)	(0.114)	(0.067)	(0.044)	(0.049)
Kalman gain (K)	0.033	0.101	0.65	0.187	0.356	0.076	0.501	0.04
diagnostic K	0.07	0.238	1.237	0.372	0.703	0.21	0.86	0.1

This table shows the mean value and standard deviation of estimated coefficients. Standard deviation is inside the brackets. Kalman gain is calculated by $K \equiv (a^2\sigma_f^2 + \sigma_\eta^2)/(a^2\sigma_f^2 + \sigma_\eta^2 + \sigma_\epsilon^2)$ and diagnostic Kalman gain (diagnostic K) is calculated by $(1 + \theta)K$. If Kalman gain or diagnostic Kalman gain is over one, investors overreact observed signals.

Table 18. Pairwise t-test of estimated θ for US EPS analysis.

	all sample	1988	1993	1998	2003	2008	2013	2018
all	0.0							
	(1.0)							
1988	0.21	0.0						
	(0.136)	(1.0)						
1993	-0.25	-0.46	0.0					
	(0.091)	(0.003)	(1.0)					
1998	-0.16	-0.37	0.09	0.0				
	(0.237)	(0.008)	(0.528)	(1.0)				
2003	-0.18	-0.39	0.07	-0.02	0.0			
	(0.259)	(0.017)	(0.672)	(0.897)	(1.0)			
2008	0.62	0.4	0.87	0.78	0.8	0.0		
	(0.0)	(0.007)	(0.0)	(0.0)	(0.0)	(1.0)		
2013	-0.43	-0.65	-0.18	-0.28	-0.26	-1.05	0.0	
	(0.001)	(0.0)	(0.171)	(0.026)	(0.087)	(0.0)	(1.0)	
2018	0.34	0.13	0.59	0.5	0.52	-0.28	0.78	0.0
	(0.024)	(0.4)	(0.0)	(0.001)	(0.003)	(0.075)	(0.0)	(1.0)

This table shows the difference of mean value of θ in each subsample and p value of t-tests in parentheses. Null hypothesis is that both subsamples have same mean value.

Table 19. Estimated parameters of time-varying severity model of US EPS analysis.

	all sample	1988	1993	1998	2003	2008	2013	2018
c	0.457	0.447	0.52	0.51	0.453	0.513	0.55	0.56
	(0.23)	(0.293)	(0.284)	(0.27)	(0.247)	(0.276)	(0.29)	(0.276)
σ_ξ	0.075	0.188	0.16	0.057	0.127	0.055	0.117	0.07
	(0.032)	(0.105)	(0.11)	(0.017)	(0.093)	(0.015)	(0.071)	(0.028)
a	1.0	1.0	1.0	1.0	0.94	1.0	0.917	1.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.05)	(0.0)	(0.038)	(0.0)
b	0.28	0.393	0.643	0.147	0.647	0.263	0.443	0.187
	(0.116)	(0.235)	(0.208)	(0.057)	(0.218)	(0.085)	(0.267)	(0.09)
σ_η	0.075	0.188	0.16	0.057	0.127	0.055	0.117	0.07
	(0.032)	(0.105)	(0.11)	(0.017)	(0.093)	(0.015)	(0.071)	(0.028)
σ_ϵ	0.303	0.28	0.138	0.322	0.238	0.295	0.19	0.352
	(0.096)	(0.095)	(0.084)	(0.058)	(0.098)	(0.055)	(0.109)	(0.061)
Kalman gain(K)	0.062	0.32	0.574	0.033	0.226	0.036	0.278	0.042

This table shows the mean value and standard deviation of estimated coefficients obtained by SMM with time-varying θ model. Standard deviation is inside the brackets.

Table 20. Pairwise t-test of estimated c for US EPS analysis.

	all	1988	1993	1998	2003	2008	2013	2018
all	0.0							
	(1.0)							
1988	-0.01	0.0						
	(0.884)	(1.0)						
1993	0.06	0.07	0.0					
	(0.347)	(0.33)	(1.0)					
1998	0.05	0.06	-0.01	0.0				
	(0.413)	(0.387)	(0.889)	(1.0)				
2003	-0.0	0.01	-0.07	-0.06	0.0			
	(0.957)	(0.925)	(0.337)	(0.4)	(1.0)			
2008	0.06	0.07	-0.01	0.0	0.06	0.0		
	(0.392)	(0.369)	(0.927)	(0.962)	(0.379)	(1.0)		
2013	0.09	0.1	0.03	0.04	0.1	0.04	0.0	
	(0.172)	(0.175)	(0.687)	(0.582)	(0.17)	(0.618)	(1.0)	
2018	0.1	0.11	0.04	0.05	0.11	0.05	0.01	0.0
	(0.121)	(0.129)	(0.583)	(0.481)	(0.121)	(0.516)	(0.892)	(1.0)

This table shows the difference of mean value of c in each subsample and p value of t-tests in parentheses. Null hypothesis is that both subsamples have same mean value.

Table 21. Pairwise t-test of estimated σ_ξ for US EPS analysis.

	all	1988	1993	1998	2003	2008	2013	2018
all	0.0							
	(1.0)							
1988	0.01	0.0						
	(0.716)	(1.0)						
1993	0.08	0.06	0.0					
	(0.038)	(0.084)	(1.0)					
1998	0.05	0.03	-0.03	0.0				
	(0.214)	(0.381)	(0.362)	(1.0)				
2003	0.04	0.03	-0.04	-0.0	0.0			
	(0.276)	(0.459)	(0.343)	(0.929)	(1.0)			
2008	0.02	0.01	-0.06	-0.02	-0.02	0.0		
	(0.529)	(0.809)	(0.108)	(0.49)	(0.579)	(1.0)		
2013	0.06	0.05	-0.02	0.01	0.02	0.04	0.0	
	(0.112)	(0.216)	(0.628)	(0.683)	(0.637)	(0.279)	(1.0)	
2018	0.06	0.05	-0.01	0.02	0.02	0.04	0.0	0.0
	(0.087)	(0.173)	(0.726)	(0.589)	(0.551)	(0.224)	(0.896)	(1.0)

This table shows the difference of mean value of σ_ξ in each subsample and p value of t-tests in parentheses. Null hypothesis is that both subsamples have same mean value.